System Design

System Performance Projection

Design Equations and Parameters

The performance of a specified RO system is defined by its feed pressure (or permeate flow, if the feed pressure is not specified) and its salt passage. In its simplest terms, the permeate flow Q through an RO membrane is directly proportional to the wetted surface area S multiplied by the net driving pressure ($\Delta P - \Delta\pi$). The proportionality constant is the membrane permeability coefficient, known as the A-value. The familiar water permeation equation has the form:

$$Q = (A)(S)(\Delta P - \Delta\pi) \qquad \text{Eq. 1}$$

The salt passage is by diffusion, hence the salt flux N_A is proportional to the salt concentration difference between both sides of the membrane. The proportionality constant is the salt diffusion coefficient, known as the B-value.

$$N_A = B(C_{fc} - C_p) \qquad \text{Eq. 2}$$

where:
C_{fc} = feed-concentrate average concentration
C_p = permeate concentration

There are basically two ways to calculate the performance of a specified design: “Element-to-Element” and “Entire System”.

Element-to-Element

This is the most rigorous calculation method. It is too tedious for hand calculation, but it is suitable for computer calculations. All the operating conditions of the first element must be known, including the feed pressure. Then the flow, pressure, etc., of the concentrate, which is the feed to the second element, can be calculated. After calculating the results for all the elements, the original feed pressure may be too high or low, so the trial and error process starts with a new pressure.

With the help of DuPont’s Water Application Value Engine (WAVE) design software, accurate results can be obtained very quickly, making it an indispensable tool for modifying and optimizing the design of an RO or an NF system. Accordingly, the entire system calculation method will not be described here. It is also not intended to outline the process of the element-to-element computer calculation. However, the governing equations and parameters are given in Table 1.

Design Equations and Parameters (Cont.)

In order to enable the determination of values for the terms A, ΔP, and $\Delta\pi$ in Eq. 1, the water permeation equation is expanded to Eq. 3. The permeate concentration can be derived from Eq. 2 after conversion into Eq. 13. The design equations are listed in Table 1, the symbol definitions in Table 3.

The subscript i in the equations of Table 1 indicates that they apply to the i^{th} element in a sequence of n elements in a series flow configuration. To accurately determine system performance, Eq. 3 is successively solved for each of the n elements starting with an inlet set of conditions. The solutions depend on mass balances around each element for salt (Eq. 7) and water (Eq. 13), as well as correlations for individual element parameters such as concentrate-side flow resistance, ΔP_{fc} (Eq. 23); temperature correction factor for water permeability, TCF (Eq. 9 and Eq. 10); polarization factor, pf_i (Eq. 11); and the membrane permeability coefficient for water, $A_i(\pi_i)$ (Eq. 24, Eq. 25, and Eq. 26), which in the case of reverse osmosis membranes depends on the average concentrate concentration or, alternatively, osmotic pressure. These solutions usually involve a suitable average for the feed and permeate side hydraulic and osmotic pressures. For low recovery values typical of single-element operation, an accurate solution can be obtained using a simple arithmetic average of the inlet and outlet conditions. Even so, since the outlet conditions are not known, iterative trial and error solutions are involved.

Design Equations and Parameters (Cont.)

Table 1: Design equations for projecting RO system performance: individual element performance

Item	Equation	Equation Number
Permeate flow	$Q_i = A_i \bar{\pi}_i S_E (\mathrm{TCF})(\mathrm{FF})\left(P_{fi} - \frac{\Delta P_{fci}}{2} - P_{pi} - \bar{\pi} + \pi_{pi}\right)$	3
Average concentrate-side osmotic pressure	$\bar{\pi} = \pi_i \left(\frac{\bar{C}_{fc}}{C_f}\right)\overline{pf}$	4
Average permeate-side osmotic pressure	$\bar{\pi}_{pi} = \pi_{fi}(1 - R_i)$	5
Ratio: arithmetic average concentrate-side to feed concentration for Element *i*	$\frac{C_{fc\,i}}{C_{f_i}} = \frac{1}{2}\left(1 + \frac{C_{ci}}{C_{f\,i}}\right)$	6
Ratio: concentrate to feed concentration for Element *i*	$\frac{C_{ci}}{C_{f\,i}} = \frac{1 - Y_i(1 - R_i)}{(1 - Y_i)}$	7
Feedwater osmotic pressure	$\pi_f = 1.12(273 + T)\sum m_j$	8
Temperature correction factor for RO and NF membrane	$\mathrm{TCF} = \mathrm{EXP}\left[2640\left(\frac{1}{298} - \frac{1}{273 + T}\right)\right]; \mathrm{T} \geq 25°\mathrm{C}$	9
	$\mathrm{TCF} = \mathrm{EXP}\left[3020\left(\frac{1}{298} - \frac{1}{273 + T}\right)\right]; \mathrm{T} \leq 25°\mathrm{C}$	10
Concentration polarization factor for 8-inch elements	$pf_i = \mathrm{EXP}[0.7Y_i]$	11
System recovery	$Y = 1 - [(1 - Y_1)(1 - Y_2)\ldots(1 - Y_n)] = 1 - \prod_{i=1}^{n}(1 - Y_i)$	12
Permeate concentration	$C_{p_j} = B(C_{fc\,j})(pf_i)(\mathrm{TCF})\frac{S_E}{Q_i}$	13

Design Equations and Parameters (Cont.)

Entire System

Average values are used to calculate feed pressure and permeate quality if the feed quality, temperature, permeate flowrate, and number of elements are known. If the feed pressure is specified instead of the number of elements, the number of elements can be calculated with a few iterations. Sample design equations for 8-inch reverse osmosis elements are listed in Table 2 and the symbols are defined in Table 3. It is recommended to use the WAVE design software for actual calculations.

Table 2: Design equations for projecting RO system performance: system average performance

Item	Equation	Equation Number
Total permeate flow	$Q = N_E S_E \bar{A} \bar{\pi}\ (TCF)(FF) \left[P_f - \frac{\overline{\Delta P_{fc}}}{2} - P_p - \pi + \pi_p \right] \left[\frac{\bar{C}_{fc}}{C_f} P_f - (\bar{1} - \bar{R}) \right]$	14
Ratio: average concentrate-side to feed concentration for system	$\frac{C_{fc}}{C_f} = \frac{-\bar{R} \ln(1 - Y/Y_L)}{Y - (1 - Y_L) \ln(1 - Y/Y_L)} + (1 - \bar{R})$	15
Limiting system recovery	$Y_L = 1 - \frac{\pi_f (\overline{pf})(\bar{R})}{P_f - \overline{\Delta P}_{fc} - P_p}$	16
Approximate log-mean concentrate-side to feed concentration ratio for system	$\left. \frac{C_{fc}}{C_f} \right\|_{Y_L, \bar{R}=1} = -\frac{\ln(1 - Y)}{Y}$	17
Average element recovery	$Y_i = 1 - (1 - Y)^{1/n}$	18
Average polarization factor	$\overline{pf} = EXP\left[0.7 \bar{Y}_i\right]$	19
Average concentrate-side osmotic pressure for system	$\pi = \pi_i \left(\frac{\bar{C}_{fc}}{C_f} \right) \overline{pf}$	20
Average concentrate-side system pressure drop for 8-inch elements; 2 stages	$\overline{\Delta P}_{fc} = 0.04 \bar{q}_{fc}^{\ 2}$	21
	$\Delta P_{fc} = \left[\frac{0.1(Q/1440)}{YN_{V2}} \right] \left(\frac{1}{N_{VR}} + 1 - Y \right)$	22
Individual 8-inch element, or single-stage concentrate-side pressure drop	$\Delta P_{fc} = 0.01 n \bar{q}_{fc}^{\ 1.7}$	23
Membrane permeability as a function of average concentrate-side osmotic pressure	$\bar{A}(\pi) = 0.125;\ \pi \le 25$	24
	$\bar{A}(\pi) = 0.125 - 0.011 \left(\frac{\pi - 25}{35} \right);\ 25 \le \pi \le 200$	25
	$\bar{A}(\pi) = 0.070 - 0.0001(\pi - 200);\ 200 \le \pi \le 400$	26
Permeate concentration	$C_p = BC_{fc} \overline{pf} (TCF) \left(\frac{N_E S_E}{Q} \right)$	27

Table 3: Symbol definitions

Q_i	permeate flow of Element *i* (gpd)	$\sum_j$	summation of all ionic species
$A_i\pi_i$	membrane permeability at 25°C for Element *i*, a function of the average concentrate-side osmotic pressure (gfd/psi)	Y	system recovery (expressed as a fraction) = permeate flow/feed flow
S_E	membrane surface area per element (ft^2)	$\prod_{i=1}^{n}$	multiplication of n terms in a series
TCF	temperature correction factor for membrane permeability	n	number of elements in series
FF	membrane fouling factor	Q	system permeate flow (gpd)
P_{f_i}	feed pressure of Element *i* (psi)	N_E	number of elements in system
ΔP_{fc_i}	concentrate-side pressure drop for Element *i* (psi)	$\overline{Q}_i$	average element permeate flow (gpd) = Q/N_E
P_{p_i}	permeate pressure of Element *i* (psi)	$\overline{A}\pi$	average membrane permeability at 25°C: a function of the average concentrate-side osmotic pressure (gfd/psi)
$\overline{\pi}_i$	average concentrate-side osmotic pressure (psi)	$\overline{C}_{fc}$	average concentrate-side concentration for system (ppm)
π_{f_i}	feed osmotic pressure of Element *i*	$\overline{R}$	average fractional salt rejection for system
π_{p_i}	permeate-side osmotic pressure of Element *i* (psi)	$\overline{\pi}$	average concentrate-side osmotic pressure for system (psi)
pf_i	concentration polarization factor for Element *i*	$\Delta\overline{P}_{fc}$	average concentrate-side system pressure drop (psi)
R_i	salt rejection fraction for Element *i* $= \frac{\text{feed conc.- perm. conc.}}{\text{feed conc.}}$	Y_L	limiting (maximum) system recovery (expressed as a fraction)
C_{fc_i}	average concentrate-side concentration for Element *i* (ppm)	$\overline{Y}_i$	average element recovery (expressed as a fraction)
C_{f_i}	feed concentration for Element *i* (ppm)	$\overline{pf}$	average concentration polarization factor
C_{c_i}	concentrate concentration for Element *i* (ppm)	$\overline{q}_{fc}$	arithmetic average concentrate-side flowrate (gpm) [= (1/2)(feed flow + concentrate flow)]
Y_i	recovery fraction for Element *i* $= \frac{\text{permeate}}{\text{feed flow}}$	N_V	number of six-element pressure vessels in system (≈ N_E/6)
π_f	treated feedwater osmotic pressure (psi)	N_{V1}	number of pressure vessels in first stage of 2-stage system (≈ 1/3 N_V)
T	feedwater temperature (°C)	N_{V2}	number of pressure vessels in second stage of 2-stage system (≈ N_V/3)
m_j	molal concentration of jth ion species	N_{VR}	stage ratio (=N_{VI}/N_{V2})

Excerpt from FilmTec™ Reverse Osmosis Membranes Technical Manual (Form No. 45-D01504-en), Chapter 3, "System Design."

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